Finite Density of States in a Mixed State of $d_{x^2-y^2} + id_{xy}$ Superconductor

W. Mao[1] and A.V. Balatsky[2]

[1] Department of Physics, Boston College, Chestnut Hill, MA 02167, USA

[2] T-Div and MST-Div, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

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We have calculated the density of states of quasiparticles in a $d_{x^2-y^2} + id_{xy}$ superconductor, and show that in the mixed state the quasiparticle spectrum remains gapless because of the Doppler shift by superflow. It was found that if the d_{xy} order gap $\Delta_1 \propto \sqrt{H}$ as suggested by experiments, then thermal conductivity $\kappa \propto \sqrt{H}$ in accord with experimental data at lowest temperatures. This is an appended version of the paper published in Phys. Rev. **B 59**, 6024, (1999). We now also discuss the disorder effects and analyze the H log H crossover at small fields. We argue that H log H regime is present and disorder effect is dominant as the field-induced seconary gap is small at small fields.

Based on the experiments recently carried out by several groups [1,2] , it has been found that the longitudinal thermal conductivity κ_{xx} of unconventional superconductor, such as $Bi_2Sr_2CaCu_2O_{8+\delta}$, displays quite strange behavior in the mixed state. With temperature from 5K to 20K, it decreases as the applied magnetic field along c-axis increases. At some critical field value $H_k(T)$, thermal conductivity becomes insensitive to the increase of magnetic field and develops a plateau. In order to explain this phenomena, it has been proposed that the $d_{x^2-y^2}$ order parameter is suppressed at H_k due to the opening of a $d_{x^2-y^2} + id_{xy}$ gap on the whole Fermi surface, thus suppressing quasiparticle contribution to thermal transport.

On the other hand, more recent measurements at lowest temperatures [2], say 0.1K, show thermal conductivity increases as field rises $\kappa_{xx} \propto \sqrt{H}$, as well as a substantial history dependence of the κ in the mixed phase. These results clearly point towards the relevance of the vortex lattice and possible disorder and hysteresis in the vortex lattice for the thermal transport in the mixed state.

From these measurements one can conclude that the spectrum of quasiparticles has a finite density of states $N(E = 0, H)$ at lowest energies. While, we argue, based on the available data for κ at lowest temperature, one still can not rule out the transition into fully gapped state, such as $d_{x^2-y^2} + id_{xy}$.

Here we present the simple model calculation which indeed shows that for fully gapped state one can still get $\kappa \propto \sqrt{H}$ if $\Delta_1 \propto \sqrt{H}$ as proposed by Laughlin [4], and can reconcile the fully gapped spectrum of quasiparticle in $d_{x^2-y^2} + id_{xy}$ state and the experimental data at lowest temperature by Aubin et al [2].

We repeated Volovik's calculation [3], in which the spectrum of quasiparticles is modified because of the Doppler shift from superflow in the mixed state. Volovik found that with the pure $d_{x^2-y^2}$ order parameter, the density of states of the delocalized quasiparticles has relation $N(E = 0, H) \propto \sqrt{H}$. Here we consider $d_{x^2-y^2} + id_{xy}$ order parameter, thus the quasiparticle spectrum is fully gapped, $E(\mathbf{k}) = \sqrt{\epsilon^2(\mathbf{k}) + |\Delta(\mathbf{k})|^2}$,

where $\Delta(\mathbf{k}) = \Delta_0(\mathbf{k}) + i\Delta_1(\mathbf{k})$, $\Delta_0(\mathbf{k})$ represents for the $d_{x^2-y^2}$ order parameter, and $\Delta_1(\mathbf{k})$ for d_{xy}. Below we assume that there is some average value of the gap $\Delta_1(\mathbf{k})$ in the vortex lattice unit cell and ignore its position dependence. The spatial variations of $\Delta_1(\mathbf{k})$ albeit substantial, will not change qualitatively results presented below.

The DOS for delocalized quasiparticles at Fermi surface can be calculated from following equation, taking $T = 0$,

$$N_{deloc}(0) = 2 \int \frac{d^3k}{(2\pi)^3} \int d^2\mathbf{r}\,\delta[E(\mathbf{k},\mathbf{r}) + m_e\mathbf{v}_F \cdot \mathbf{v}_s], \quad (1)$$

where $\mathbf{v}_s = (\hbar/2m_e)(\widehat{\phi}/r)$. The main contribution of the DOS comes from the vicinity of the $d_{x^2-y^2}$ gap nodes, the gap function has the general form around the node $\Delta_0(\mathbf{k}) \approx \vec{\gamma}(k_z) \cdot [\mathbf{k} - \mathbf{k}_n(k_z)]$, where $\vec{\gamma}(k_z) || \mathbf{e_z} \times \mathbf{k}_n$. It is convenient to introduce new momentum variables $\widetilde{k}_x = (\mathbf{k} - \mathbf{k}_n) \cdot \widehat{k}_n, \widetilde{k}_y = (\mathbf{k} - \mathbf{k}_n) \cdot \widehat{\gamma}_n$, where $\widehat{k}_n$ is the unit vector in the direction of the gap nodes in the $a - b$ plane, $\widehat{\gamma}_n$ the unit vector along the direction of $\vec{\gamma}$. Then we can write Eq.(1) as

$$N_{deloc}(0) = 4 \times \frac{1}{4\pi^3} \int d\widetilde{k}_y dk_z \frac{d\epsilon}{v(k_z)} d^2\mathbf{r}$$
$$\delta[\sqrt{\epsilon^2 + \widetilde{k}_y^2\gamma^2(k_z) + \Delta_1^2} + m_e\mathbf{v}_F(\mathbf{k}_n) \cdot \mathbf{v}_s], \quad (2)$$

and the integral can be simplified as

$$N_{deloc}(0) =$$
$$\frac{2}{\pi^2 v_F\gamma} \int dk_z \int d\theta \int_\xi^{R_H} r\,dr \int_0^{k_c{}^2} dx\,\delta(\sqrt{x + \Delta_1^2} + k_F v_s cos\theta). \quad (3)$$

Substituting $\sqrt{x + \Delta_1^2}$ by the variable y, we can further write above equation as

$$N(0) = \frac{2}{\pi^2 v_F\gamma} \int_{\Delta_1}^\infty dy \int dk_z \int d\theta \int_\xi^R r\,dr\,y\,\delta(y + k_F v_s cos\theta). \quad (4)$$

In order to find the analytical form we first express the delta function as an integral form, $\delta(y + k_F v_s cos\theta) = \int_{-\infty}^{\infty} d\lambda \exp(i\lambda y - i\lambda k_F v_s cos\theta)$, and integrate over θ, then over λ. We find from Eq.(4),

$$N(0) = \int \frac{2dk_z}{\pi v_F \gamma} \int_{\xi}^{R_H} rdr\sqrt{k_F^2 v_s^2 - \Delta_1^2}\Theta(k_F^2 v_s^2 - \Delta_1^2)$$

$$= \int dk_z \frac{2\Delta_1}{\pi v_F \gamma}\int_{\xi}^{Min(r_0,R_H)} dr\sqrt{r_0^2 - r^2}$$

$$\approx \int dk_z \frac{\Delta_1}{\pi v_F \gamma}(R_H\sqrt{r_0^2 - R_H^2} + r_0^2 arcsin\frac{R_H}{r_0}) \qquad (5)$$

where $r_0 = \frac{1}{2}\xi\widetilde{\Delta}^{-1}$, $\widetilde{\Delta} = \Delta_1/\Delta_0$, R_H is the intervortex distance, and ξ is the coherent length for pairing. We already know that $R_H \approx \xi\sqrt{H_{c2}/H}$, and assume Δ_1 much smaller compared with Δ_0, hence $Min(r_0, R_H) = R_H$. After making Taylor expansion in terms of R_H/r_0, we can get the following relation averaged over the vortices

$$N(0) \approx KN_F\widetilde{\Delta}(\frac{r_0}{R_H} - \frac{1}{3}\frac{R_H}{r_0}) \sim \sqrt{H} - \frac{\Delta_1^2}{\sqrt{H}}. \qquad (6)$$

Here the factor K is on the order of unity. We found that the spectrum remains gapless. The origin of the gapless behavior is the Doppler shift of quasiparticle states $E - \mathbf{k}\mathbf{v}_s$, which is position dependent. There are regions where this shift is larger than minimal gap in the spectrum Δ_1, thus leading to the finite DOS. This point was also emphasized by Hirschfeld and Wölfle, who showed that for finite id' gap the density of states will be finite [5]. We note that regardless of the power with which $\Delta_1 \sim H^\alpha$ opens up in the field, as long as $\alpha \geq 1/2$, the leading term in DOS will always be $\sqrt{H}$ at small fields.

If one uses the form of $\Delta_1 = \hbar v\sqrt{2eH/(\hbar c)}$ used by Laughlin [4] (v is the average quasiparticle velocity in the d-wave node) to explain Krishana's data, we find that for the DOS of the fully gapped state

$$N(E = 0, H) = KN_F(\sqrt{\frac{H}{H_{c2}}} - \frac{4}{3}\frac{\hbar e v^2}{c\Delta_0^2}\sqrt{H_{c2}H})$$

$$= KN_F\sqrt{\frac{H}{H_{c2}}}(1 - \frac{4\Delta_0}{3E_F}), \qquad (7)$$

where E_F is the Fermi energy. Typically $\Delta_0/E_F \sim 10^{-1}$ and the effect of induced gap is to lower the $\sqrt{H}$ prefactor in $N(0)$ by about 20%. The DOS with $N(E = 0, H) \sim \sqrt{H}$ was previously thought to be characteristic of "pure" $d_{x^2-y^2}$ state [3].

Experimental result by Aubin et al. [2] showed that the thermal conductivity at lowest temperature with $\kappa(H) \propto \sqrt{H}$ is consistent with quasiparticle transport and "Volovik effect" in the density of states. They also pointed out that even at lowest temperature data are consistent with the gapped $d_{x^2-y^2} + id_{xy}$ phase provided Δ_1 is small enough.
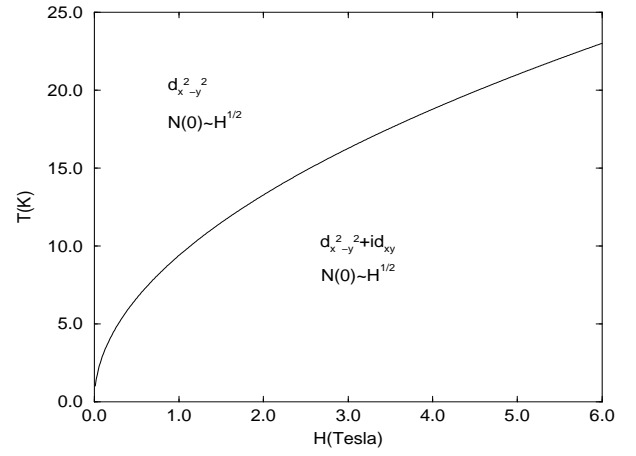

FIG. 1. Two regions of the phase diagram are shown. The line is the the boundary between gapped and gapless states, given by $k_B T_c = 0.52\hbar v\sqrt{\frac{2eH}{\hbar c}}$. In both phases the density of states $N(0, H) \propto \sqrt{H}$ due to $\Delta_{xy} \propto \sqrt{H}$.

Fig.1 demonstrates that the superconductor has two phases [4] as temperature and applied magnetic field change, one with "pure" $d_{x^2-y^2}$ order parameter, the other with $d_{x^2-y^2} + id_{xy}$ order parameter. According to our calculation, in both phases, the DOS of quasiparticle at $E = 0$ is proportional to square root of applied field.

We also note the implication of our calculation for the specific heat. Recent measurements of the specific heat $C(T, H)$ in the mixed state of YBCO superconductors by Moler et.al. [6] indicate that the low temperature density of states indeed scales as $N(E = 0, H) \sim \sqrt{H}$. This observation however, as we argue, is not inconsistent with the $d + id'$ gap in the mixed state.

In the paper by Volovik [3], it was mentioned that in the mixed state of the d-wave superconductor, the conventional s-wave pairing will be generated in the core of vortex. This might be another possibility to obtain fully gapped quasiparticle spectrum. However, it will not affect our result which comes from the region outside the core.

The main result of this work is to show that even in the presence of fully gapped spectrum, the superflow modifies quasiparticle spectrum and keeps DOS gapless in the mixed state. Approximating thermal conductivity as proportional to the DOS at $E = 0$ at lowest temperature, and assuming that the magnitude of Δ_1 proportional to $\sqrt{H}$ in accordance with the work by Laughlin, one can still be consistent with experimental data at lowest temperature. It shows that we cannot rule out the possibility that a second superconducting phase appears in the magnetic field at low temperatures.

Note Added (March, 1999): In the paper by C. Kubert and P.J. Hirschfeld [7], the density of states was calculated in a $d_{x^2-y^2}$ superconductor in the dirty limit, and they found a $H\ln H$ behavior at the lowest field instead of $\sqrt{H}$. Here we also calculated the density of states with the induced gap Δ_1 in the presence of impurity scatter-

ing, and found that the appearance of the induced gap Δ_1 does not change the $HlogH$ behavior in the lowest field. The details of calculation are described in the appendix.

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1. Appendix A

We assume the impurity gives an imaginary term γ_0 in the self energy of one particle Green function. Thus the density of states can be calculated by following equation:

$$
N(E=0,H) = \frac{1}{\pi R_H^2} \int d^3\mathbf{k} \int d^2\mathbf{r} Im(G)
$$

$$
= \frac{1}{\pi R_H^2} \int d^3\mathbf{k} \int d^2\mathbf{r} \frac{\gamma_0}{(\mathbf{v}\cdot\mathbf{k}+\sqrt{\epsilon_k^2+|\Delta|^2})^2+\gamma_0^2}
$$

$$
= \frac{1}{R_H^2\gamma v_F} \int rdr \int d\theta \int dk_z \int dx \frac{\gamma_0}{(\mathbf{v}\cdot\mathbf{k_F}+\sqrt{x+\Delta_1^2})^2+\gamma_0^2}
$$

$$
= \frac{\gamma_0}{R_H^2\gamma v_F} \int rdr \int d\theta \int dk_z \int_{\Delta_1}^{Y} \frac{ydy}{(\mathbf{v}\cdot\mathbf{k}+y)^2+\gamma_0^2}
$$

$$
= \frac{\gamma_0}{\gamma v_F} \int dk_z \int_{\frac{\xi}{R_H}}^{1} xdx \int d\theta \{ \frac{E_H}{\gamma_0} \frac{cos\theta}{x}[arctan(\frac{\frac{E_H cos\theta}{x}+\Delta_1}{\gamma_0})
$$

$$
-\frac{\pi}{2}] + \frac{1}{2}ln\frac{(\frac{E_H cos\theta}{x}+Y)^2+\gamma_0^2}{(\frac{E_H cos\theta}{x}+\Delta_1)^2+\gamma_0^2} \},
$$

where $E_H = \sqrt{H/H_{c2}}\Delta_0$, Y is an upper cutoff of $\sqrt{\epsilon_k^2+|\Delta|^2}$. It should be noticed that γ_0 is the imaginary self energy term from impurity, while γ is the gradient of Δ_0 with respect to $\widetilde{k}_y$

In above equation, we can get the $HlogH$ behavior when Δ_1 equals to zero. We calculated the integral numerically, including both the effect of induced d_{xy} gap and impurity, and show them in Fig.2. In our calculation we used the result, $\gamma = 0.61\sqrt{\Gamma\Delta_0}$, of Kubert and Hirschfeld [7], with $\Gamma = n_i/\pi N_F$. It is found that the presence of an induced d_{xy} gap still cannot change the behavior of the density of states as a function of applied field, however it decreases the density of states a little bit.
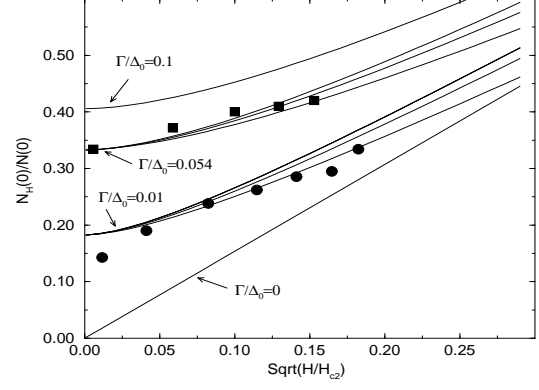

FIG. 2. Normalized density of states vs $(H/H_{c2})^{1/2}$. The three line groups for each data sets correspond to different strength of $\Delta_1 = x\Delta_0(H/H_{c2})^{1/2}$ with $x = 0, 0.5, 0.8$ from top line to bottom. The presence of the *field induced* $\Delta_1 \propto H^{1/2}$ does not change the $N_H(0)$ behavior at lowest fields as the effect of disorder and Δ_0 dominates this region. Hence for dirty $d_{x^2-y^2}+id_{xy}$ we expect the same $H^{1/2} \to H \ln H$ crossover, as discussed before [7]. The data are from references [6] and [8].

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